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Bose-Einstein correlations and the equation of state of nuclear matter

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Abstract. Within a relativistic hydrodynamic framework, we use four different equations of state of nuclear matter to compare to experimental spectra from CERN/SPS experiments NA44 and NA49. Freeze-out hypersurfaces and Bose-Einstein correlation functions for identical pion pairs are discussed. We find that two-pion Bose-Einstein interferometry measures the relationship between the temperature and the energy density in the equation of state during the late hadronic stage of the fireball expansion. Little sensitivity of the light-hadron data to a quark-gluon plasma phase-transition is seen.

The equation of state (EOS) of nuclear matter at very high energy densities remains unknown. A possible new state of nuclear matter, the quark-gluon plasma (QGP), may be formed within a very hot and dense zone of nuclear matter - the fireball - during relativistic heavy-ion collision experiments. Many observables have been proposed as a signature for the QGP, among them measurements derived from intensity interferometry of identical hadrons, also known as Bose-Einstein correlations (BEC). BEC functions are sensitive to the space-time dynamics of the fireball, and therefore should give clues about the EOS, which governs the evolution of those fireballs.

Among the many studies on BEC (for a recent overview, *cf.*, the book by Weiner [1]) only few (*cf.*, e.g., Rischke *et al.* [2]) address the role of the EOS of nuclear matter in the intensity interferometry of identical hadrons. It is the purpose of this paper to further investigate the interplay between the EOS and BEC. In the following, we shall consider a framework of analysis which is based on relativistic hydrodynamics, since this approach allows for an explicit use of an EOS.

We shall use the simulation code HYLANDER-C [3] with various equations of state, and Cooper-Frye [4] freeze-out. The results of these hydrodynamical (one-fluid-type) calculations [5] will be compared to experimental single particle momentum distributions from 158A GeV Pb+Pb collisions, measured by the NA44 [6,7] and NA49 [8] Collaborations. After a discussion of the space-time features of the particular fireballs we shall compare BEC of identical pion pairs [9] to experimental correlation functions, as measured by NA44 [10]. Comparison with data of other experiments is possible (and would not alter our conclusions), but for the sake of *illustration* we restrict ourselves to the use of NA44's BEC functions.

The first equation of state [11], EOS-I, which we use in this study exhibits a phase transition to a quark-gluon plasma at a critical temperature $T_c = 200\ MeV$ with a critical energy density $\epsilon_c = 3.2\ GeV/fm^3$. The second equation of state [12], EOS-II, is a lattice QCD-based EOS (as is EOS-I) which has recently become very popular in the field of relativistic heavy-ion physics. This equation of state includes a phase transition to a quark-qluon plasma at $T_c = 160\ MeV$ with a critical energy density $\epsilon_c \approx 1.5\ GeV/fm^3$. The third equation of state, EOS-III, has been extracted from the microscopic transport model RQMD [13] under the assumption of complete thermalization, and does *not* include a transition to a QGP. We obtain a fourth equation of state, EOS-Ib, by changing the relationship between ϵ_c and T_c in EOS-I to $T_c(\epsilon_c = 1.35\,GeV/fm^3) = 200\ MeV$.

In Fig. 1 the four equations of state are shown in two different representations. The particular parametrizations for the EOS which have been used here are described in [5] in more detail.

In the following, we shall discuss five scenarios: we compare four calculations using EOS-I, EOS-II, EOS-III, and EOS-Ib, all at fixed freeze-out temperature $T_f = 139\ MeV$, and one calculation using EOS-I for fixed freeze-out temperature $T_f = 116\ MeV$.

It is possible to find initial distributions (*cf.* Table 2 in [5] and Table 1 (here)) for the four equations of state, such that one can reproduce the single inclusive momentum spectra of 158 $AGeV$ Pb+Pb collisions. For a freeze-out temperature $T_f = 139\ MeV$, the initial conditions and a large number of final single inclusive momentum distributions for various hadron species have been shown in [3, 5] in comparison to the data measured by the NA44 [6,7] and NA49 [8] Collaborations. Those results refer to calcu-

Table 1. Properties of the fireballs

	EOS-I	EOS-I	EOS-Ib
	Initial parameters		
Rel. fraction of thermal energy in the central fireball, K_L	0.55	0.60	0.60
Longitudinal extension of the central fireball, Δ $[fm]$	1.20	1.20	1.20
Rapidity at the edge of the central fireball, y_Δ	1.00	1.05	1.00
Rapidity at maximum of initial baryon y distribution, y_m	0.80	0.85	0.85
Width of initial baryon y distribution, σ	0.32	0.32	0.32
Freeze-out temperature, T_f $[MeV]$	139	116	139
	Output		
Max. initial energy density, ϵ_Δ $[GeV/fm^3]$	15.3	16.8	16.8
Max. initial baryon density, n_B^{max} $[fm^{-3}]$	3.93	4.11	3.90
Rel. fraction of baryons in central fireball, $f_{n_B}^\Delta$	0.71	0.78	0.73
Freeze-out energy density, ϵ_f $[GeV/fm^3]$	0.292	0.127	0.127
Max. transverse velocity at freeze-out, $v_\perp^{max}$ $[c]$	0.46	0.61	0.58
Lifetime of fireball, t_{max} $[fm/c]$	13.1	20.2	20.5
Lifetime of QGP, t_{QGP} $[fm/c]$	2.4	2.6	6.0

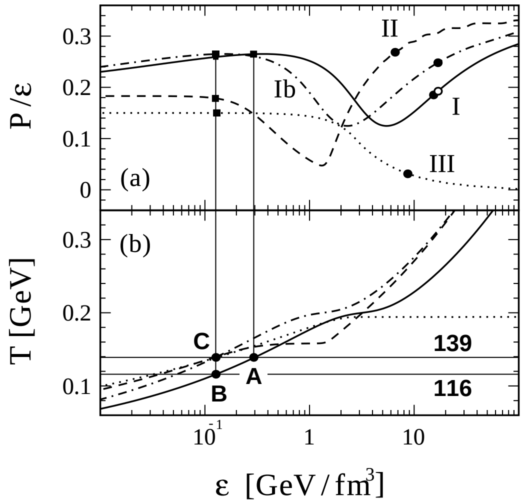

Fig. 1. Ratio of pressure and energy density, P/ϵ, and temperature, T, as functions of ϵ, for the equations of state EOS-I (solid lines), EOS-II (dashed lines), EOS-III (dotted lines), and EOS Ib (dashed-dotted lines), respectively. The dots in plot (b) correspond for each EOS to the starting values of P/ϵ with respect to the achieved initial maximum energy density ϵ_Δ at transverse position $r_\perp = 0$, (dots correspond to $T_f = 139$ MeV whereas the open circle corresponds to $T_f = 116$ MeV). The squares indicate the final values of P/ϵ at breakup energy densities, ϵ_f. The dots A, B, C in plot (c) indicate the relationship between the temperature and the energy density at the late hadronic stage of the fireball expansion

lations using EOS-I, EOS-II, and EOS-III. Results showing the fits of single inclusive particle momentum spectra using EOS-Ib with $T_f = 139$ MeV and EOS-I with $T_f = 116$ MeV and $T_f = 139$ MeV are shown in Fig. 2.

The magnitudes of the slopes in the transverse mass spectra have their origin in the freezeout temperature and the transverse velocity fields at freeze-out. Because the effective EOS softness [5] is larger in the calculations using EOS-Ib with $T_f = 139$ MeV and EOS-I with $T_f = 116$ MeV compared to the calculation using EOS-I with $T_f = 139$ MeV, the values for the maximum transverse velocity at freeze-out, $v_\perp^{max}$, are correspondingly larger (*cf.* Table 1) in the calculations using EOS-Ib with $T_f = 139$ MeV and EOS-I with $T_f = 116$ MeV compared to the calculation using EOS-I with $T_f = 139$ MeV. But because in the calculation using EOS-I with $T_f = 116$ MeV the temperature is decreased while increasing the value for $v_\perp^{max}$, hardly any change is seen in the slopes of the transverse mass spectra compared to the calculation using EOS-I with $T_f = 139$ MeV. A more pronounced change is seen only in the calculation using the harder EOS-Ib while keeping T_f unchanged.

It should be stressed that all calculations discussed so far (except the calculation using EOS-II) result in single particle momentum distributions that describe the data equally well. Although EOS-II was found in the calculations of hadronic transverse mass spectra to be too soft (*cf.* [3,5]), we shall use it here also for the calculation of Bose-Einstein correlation functions.

Before we discuss BEC, we briefly discuss the thermal evolution of the various fireballs.

Figure 3 shows the isothermes for the relativistic Pb+Pb fluids governed by EOS-I and EOS-Ib (*cf.* also [5]) until freeze-out has been reached. In [5] it was shown that the calculation using EOS-I with $T_f = 139$ MeV leads to a fireball of a much shorter liftime than the calculations using EOS-II and EOS-III with $T_f = 139$ MeV. This behavior is caused by much smaller freeze-out energy densities, ϵ_f, in the calculations using EOS-II and EOS-III compared to the calculation using EOS-I. We have $\epsilon_f = 0.292$ [0.126 (0.130)] GeV/fm^3 when using EOS-I [EOS-II (EOS-III)]. A fluid that undergoes adiabatic expansion needs more time to reach the smaller freeze-out energy densities.

Since EOS-II and EOS-III yield similar lifetimes of the fireball, we attempt in the following to increase the lifetime of the system which is governed by EOS-I. This can be achieved by (a) using a smaller freeze-out temperature $T_f = 116$ MeV, or (b) by hardening the EOS, i.e., using EOS-Ib instead of EOS-I (without changing $T_f = 139$ MeV). For the latter two cases, we obtain $\epsilon_f = 0.127$ GeV/fm^3 (*cf.* Fig. 1 (c) and [9]).

The lifetime of the fireball is reflected in Bose-Einstein correlations of identical pion pairs [14]. Using the Bertsch-Pratt parametrization [15], a sensitive quantity is the lon-

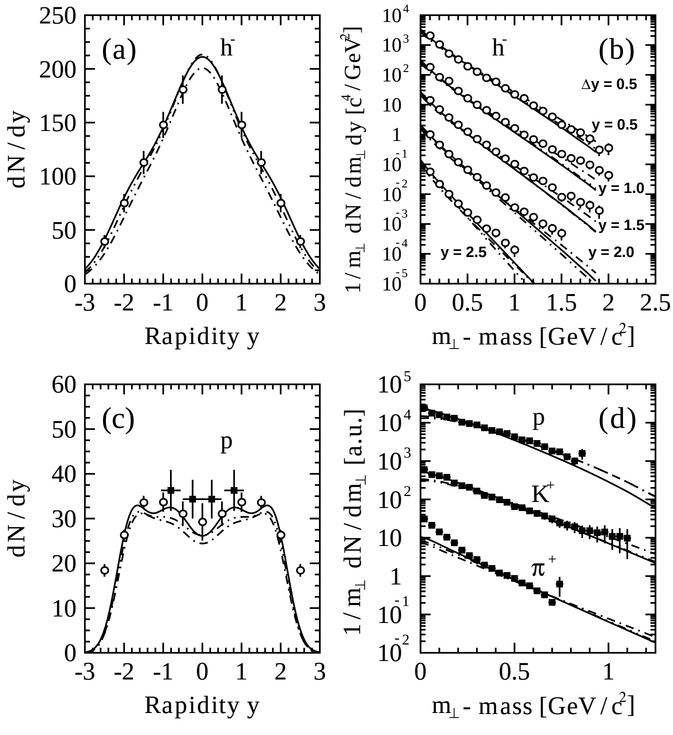

Fig. 2. **a** Rapidity spectra and **b** transverse mass spectra, $1/m_{\perp}dN/dm_{\perp}dy$, of negative hadrons, h^-, **c** rapidity spectra of protons (without contributions from Λ^0 decay) and **d** transverse mass spectra, $1/m_{\perp}dN/dm_{\perp}$, of protons (including contributions from Λ^0 decay), p, positive kaons, K^+, and positive pions, π^+, respectively. The solid (double dotted-dashed [dashed-dotted]) lines indicate the results of the calculations when using equation of state EOS-I (EOS-I [EOS-Ib]) with T_f = 139 (116 [139]) MeV. The open circles represent preliminary data taken by the NA49 Collaboration, whereas the filled squares represent final data taken by the NA44 Collaboration

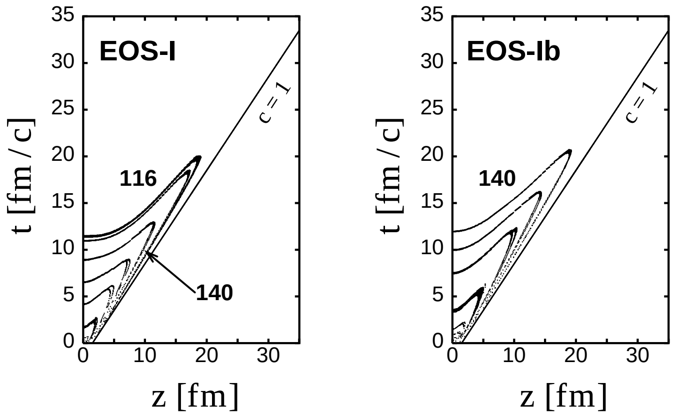

Fig. 3. Isothermes for the relativistic fluids governed by EOS-I, and EOS-Ib at $r_{\perp} = 0$, respectively. For EOS-I the lines (beginning with the most outer curves) correspond to temperatures, $T = 116$ MeV, 120 MeV, 140 MeV, 160 MeV ... etc. For EOS-Ib, the outer lines correspond to a temperature, T = 140 MeV, and each successively smaller curve represents a reduction in temperature by $\Delta T = 20$ MeV. The lines $c = 1$ represent the light cone

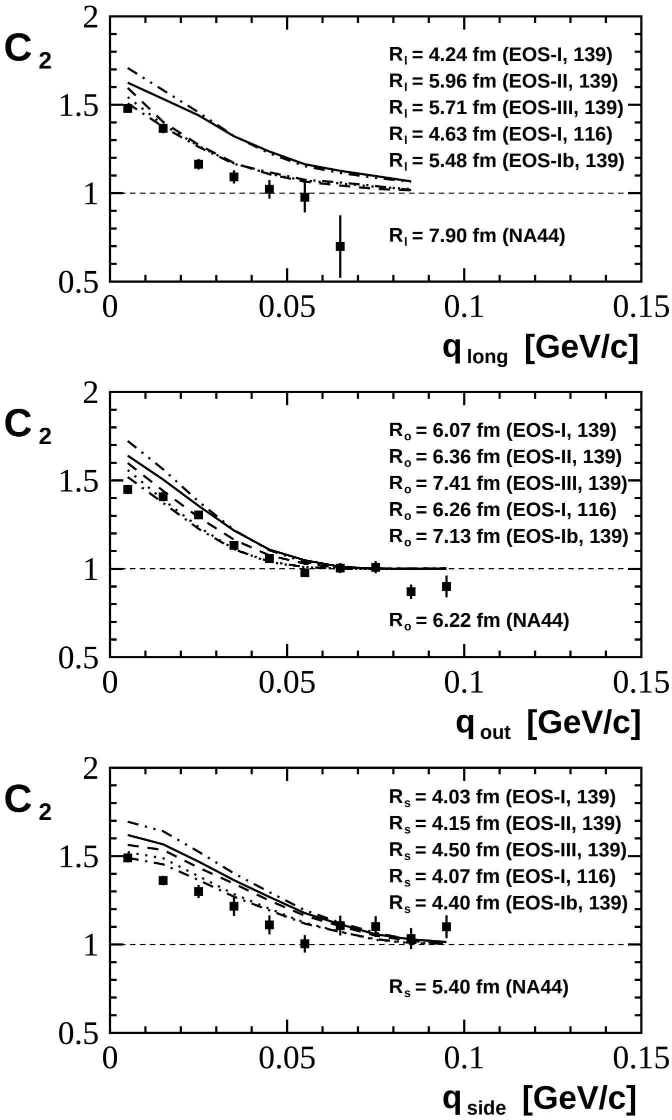

Fig. 4. Projections of BEC functions for $\pi^+\pi^+$ pairs emerging from the low $p_{\perp}$ (horizontal and vertical) acceptance setting of the NA44 detector [8]. The data points are data taken by the NA44 Collaboration [8]. The solid [dashed [dotted [dashed-dotted]]] lines correspond to the calculations using EOS-I [EOS-II [EOS-III [EOS-Ib]]] with $T_f = 139$ MeV, and the double dotted-dashed lines correspond to the calculation using EOS-I with $T_f = 116$ MeV. The values of the inverse width parameters, R_i $(i = l, o, s)$, have been obtained from the Gaussian Bertsch-Pratt parametrization (see text)

gitudinal projection of the two-pion correlation function, $C_2(q_{long})$, since it reflects the effects of transverse expansion [16] as well as the contributions of resonance decay [17] in BEC.

Figure 4 shows data points taken by the NA44 Collaboration [10], along with projections of calculated BEC functions for $\pi^+\pi^+$ pairs, using the same acceptance as the experiment. Consistent with expectation, the calculations using EOS-II, EOS-III, and EOS-Ib give similar lifetimes and therefore sufficiently large longitudinally expanded fireballs (see Figs. 2,3). These calculations result in a good reproduction of the pionic NA44 BEC data. In addition, EOS-III and EOS-Ib produce an excellent description [3,5,9] of hadronic single inclusive momentum spectra.

It should be stressed here that a freeze-out temperature $T_f = 139\ MeV$ was *adequate* to achieve this agreement.

Also consistent with expectation is that the calculation using EOS-I with $T_f = 139\ MeV$ gave a longitudinally expanded fireball [5] which was too small. On the contrary, it is initially surprising that a reduction of the freeze-out temperature to $T_f = 116\ MeV$ does not lead to a large enough longitudinal extension of the fireball in the calculation which uses EOS-I. The reason for this result is the following: a freeze-out temperature reduction leads to a larger lifetime of the *direct* fireball, but because of the lower temperature, the relative fraction of heavy resonance decay contributions is reduced (by about 30%), so that the 'resonance halo' is reduced in size. Hence, the *apparent* fireball, which is a superposition of the direct (or thermal) fireball and the resonance halo remains more or less unchanged in size.

We note that the transverse projections of the correlation functions $C_2(q_{out})$ and $C_2(q_{side})$ are also described reasonably well, especially by those calculations which yield the larger *longitudinal* freeze-out extension. The numbers in Fig. 4 can be obtained from a 1-dimensional fit of the correlation functions with [18]

$$C_2(\vec{k}_1, \vec{k}_2) = 1 + \lambda(\vec{K}) \cdot \exp[-q_l^2 R_l^2(\vec{K}) - q_o^2 R_o^2(\vec{K}) - q_s^2 R_s^2(\vec{K}) + 2\rho_{ol}(\vec{K}) q_o q_l R_o(\vec{K}) R_l(\vec{K})], \quad (1)$$

but they should not be taken too seriously. In (1) the q_i $(i = l, s, o)$ refer to the components of the momentum difference $\vec{q} = \vec{k}_1 - \vec{k}_2$, and $\vec{K} = \frac{1}{2}(\vec{k}_1 + \vec{k}_2)$ is the average pion pair momentum. Furthermore, $\lambda(\vec{K})$ is the momentum dependent incoherence factor which accounts for reductions of the BEC due to long-lived resonances [17] and averaging due to phase-space, respectively.

In conclusion, by inspecting Fig. 1(b) we can see that only those equations of state which go through point C in Fig. 1 reproduce the experimental data on Bose-Einstein correlations fairly well. Changing the freeze-out temperature shows hardly any effect. Therefore, the measurements of Bose-Einstein correlations tell us which relationship between temperature and energy density is neccessary for a valid choice of an equation of state in the calculations. Unfortunately, from the above considerations it must be noted that a two-particle BEC *used by itself* cannot be used as a tool to determine a possible phase-transition to a QGP, because the BEC show little sensitivity to the structure of the EOS. We stress, that our conclusions are based on the use of Cooper-Frye [4] freeze-out. We believe that our conclusions are all valid, if we stay within one and the same theoretical framework of analysis.

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